

On the method for calculating the trace formulas of the fractional Sturm-Liouville operator

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The trace formulas of linear differential operators deeply reveal the structure of the spectrum of differential operators and have important applications in the numerical calculation of eigenvalues, the theory of inverse problems, and the theory of solitons. For the first time in work [1], a regularized trace for the Sturm-Liouville operator is calculated. This result was generalized by many authors, and methods were developed for generalizing this formula to the case of other operators [2-4]. The trace formulas for the conformable fractional Sturm-Liouville problem was studied in [5].

In this paper, we present a new approach for obtaining the trace formula for the conformable fractional Sturm-Liouville problem.

We consider the conformable fractional Sturm-Liouville problem

$$\begin{cases} l_{\alpha} y := -D_x^{\alpha} D_x^{\alpha} y + q(x)y = \lambda y \\ D_x^{\alpha} y(0) - hy(0) = 0 \\ D_x^{\alpha} y(\pi) - y(\pi) = 0 \end{cases} \quad 0 < x < \pi$$

where λ is a spectral parameter and $q(x)$ is a real continuous function. In (1), $D_x^{\alpha} y$ denotes the fractional derivative defined as follows:

Definition. Let $f : [0, \infty) \rightarrow R$ be a given function. Then, the conformable fractional derivative of f of order α is defined by:

$$D^{\alpha} f(x) = \lim_{h \rightarrow 0} \frac{f(x + hx^{1-\alpha}) - f(x)}{h}, \quad (1)$$

for all $x > 0$, $\alpha \in (0, 1]$.

For more information about conformable fractional derivative see [6-7].

Let consider the following (1) Dirichlet boundary value problem

Theorem. The following trace formula is hold

$$\sum_{n=0}^{\infty} \left[\lambda_n - \frac{\alpha^2}{\pi^{\alpha}} n^2 - c_0 \right] = \frac{q(0) + q(\pi)}{4} - \frac{1}{2} c_0 - \frac{\alpha}{\pi^{\alpha}} (h + H) - \frac{1}{2} (h + H)^2$$

Where

$$c_0 = \frac{\alpha}{\pi^\alpha} \int_0^\pi q(t) d_\alpha t$$

References

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